TIME-CONSISTENCY: FROM OPTIMIZATION TO RISK MEASURES

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Resumen

Stochastic optimal control is concerned with sequential decision-making under uncertainty. The theory of dynamic risk measures gives values to stochastic processes (costs) as time goes on and information accumulates. Both theories coin, under the same vocable of *time-consistency* (or *dynamic-consistency*), two different notions: the latter is consistency between successive evaluations of a stochastic processes by a dynamic risk measure as information accumulates (a form of monotonicity); the former is consistency between solutions to intertemporal stochastic optimization problems as information accumulates. Interestingly, time-consistency in stochastic optimal control and time-consistency for dynamic risk measures meet in their use of dynamic programming, or nested, equations. We provide a theoretical framework that offers i) basic ingredients to jointly define dynamic risk measures and corresponding intertemporal stochastic optimization problems ii) common sets of assumptions that lead to time-consistency for both. Our theoretical framework highlights the role of time and risk preferences, materialized in one-step aggregators, in time-consistency. Depending on how you move from one-step time and risk preferences to intertemporal time and risk preferences, and depending on their compatibility (commutation), you will or will not observe time-consistency. We also shed light on the relevance of information structure by giving an explicit role to a state control dynamical system, with a state that parameterizes risk measures and is the input to optimal policies.